Latihan Soal Pembuktian Induksi

3. Let $P(n)$ be the statement that $1^2 + 2^2 + \cdots + n^2 = n(n + 1)(2n + 1)/6$ for the positive integer n.

a) What is the statement $P(1)$?

b) Show that $P(1)$ is true, completing the basis step of a proof that $P(n)$ is true for all positive integers n.

c) What is the inductive hypothesis of a proof that $P(n)$ is true for all positive integers n?

d) What do you need to prove in the inductive step of a proof that $P(n)$ is true for all positive integers n?

e) Complete the inductive step of a proof that $P(n)$ is true for all positive integers n, identifying where you use the inductive hypothesis.

f) Explain why these steps show that this formula is true whenever n is a positive integer.

5. Prove that $1^2 + 3^2 + 5^2 + \cdots + (2n + 1)^2 = (n + 1)(2n + 1)(2n + 3)/3$ whenever n is a nonnegative integer.

7. Prove that $3 + 3 \cdot 5 + 3 \cdot 5^2 + \cdots + 3 \cdot 5^n = 3(5^{n+1} - 1)/4$ whenever n is a nonnegative integer.

19. Let $P(n)$ be the statement that

$$1 + \frac{1}{4} + \frac{1}{9} + \cdots + \frac{1}{n^2} < 2 - \frac{1}{n},$$

where n is an integer greater than 1.

Complete the inductive step of a proof by mathematical induction that $P(n)$ is true for all integers n greater than 1.

7. Give a recursive definition of the sequence $\{a_n\}$, $n = 1, 2, 3, \ldots$ if

a) $a_n = 6n$.
b) $a_n = 2n + 1$.
c) $a_n = 10^n$.
d) $a_n = 5$.

11. Give a recursive definition of $P_m(n)$, the product of the integer m and the nonnegative integer n.

23. Give a recursive definition of the set of positive integers that are multiples of 5.

25. Give a recursive definition of

a) the set of even integers.
b) the set of positive integers congruent to 2 modulo 3.
c) the set of positive integers not divisible by 5.