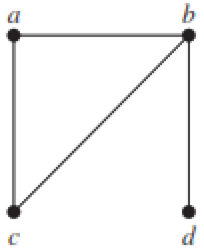
Latihan Soal Graph

For Exercises 3–9, determine whether the graph shown has directed or undirected edges, whether it has multiple edges and whether it has one or more loops. Use your answers to determine the type of graph in Table 1 this graph is.

3.



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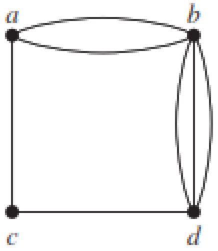

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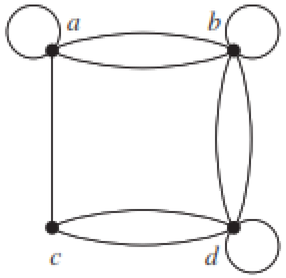

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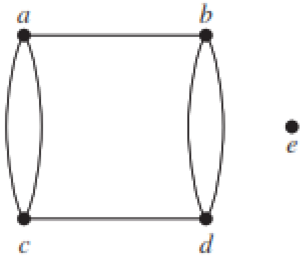

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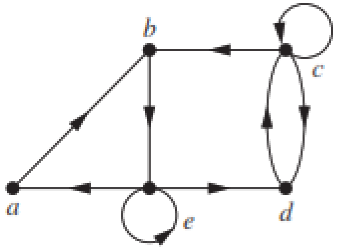

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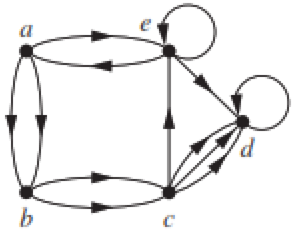

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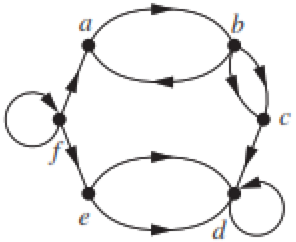
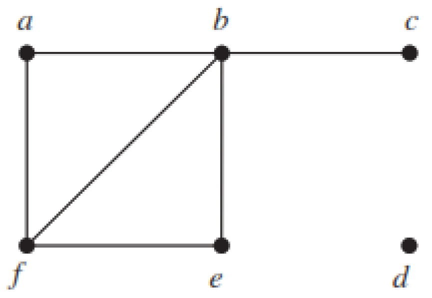
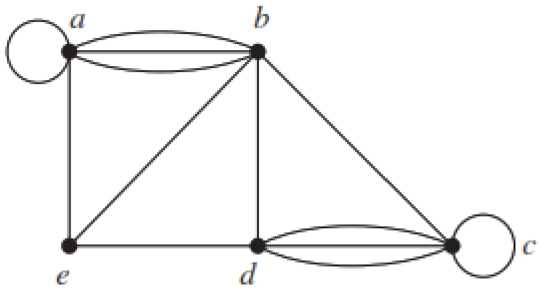


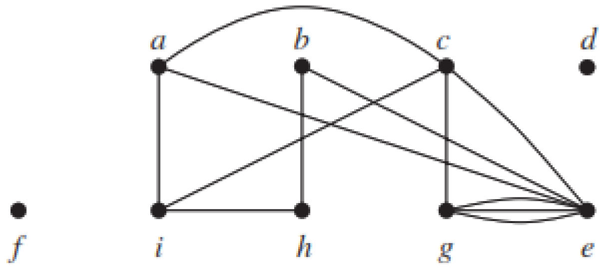
TABLE 1 Graph Terminology.

Type	*Edges*	*Multiple Edges Allowed?*	*Loops Allowed?*
Simple graph	Undirected	No	No
Multigraph	Undirected	Yes	No
Pseudograph	Undirected	Yes	Yes
Simple directed graph	Directed	No	No
Directed multigraph	Directed	Yes	Yes
Mixed graph	Directed and undirected	Yes	Yes

In Exercises 1–3 find the number of vertices, the number of edges, and the degree of each vertex in the given undirected graph. Identify all isolated and pendant vertices.

1. 

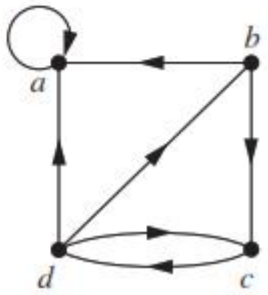
2. 

3. 

4. Find the sum of the degrees of the vertices of each graph in Exercises 1–3 and verify that it equals twice the number of edges in the graph.

In Exercises 7–9 determine the number of vertices and edges and find the in-degree and out-degree of each vertex for the given directed multigraph.

7.



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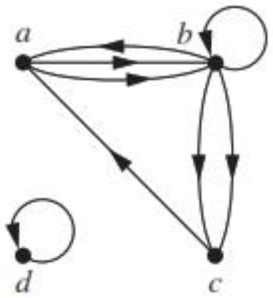

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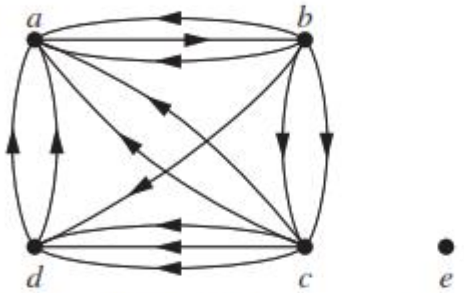
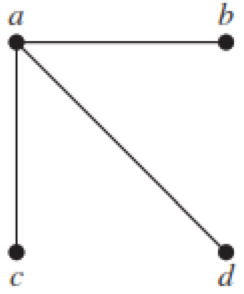


10. For each of the graphs in Exercises 7–9 determine the sum of the in-degrees of the vertices and the sum of the out-degrees of the vertices directly. Show that they are both equal to the number of edges in the graph.

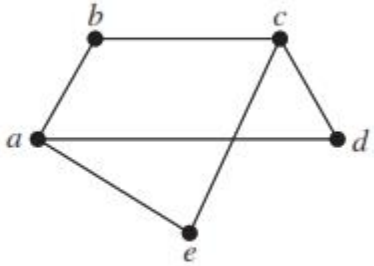
53. Draw all subgraphs of this graph.



In Exercises 21–25 determine whether the graph is bipartite. You may find it useful to apply Theorem 4 and answer the question by determining whether it is possible to assign either red or blue to each vertex so that no two adjacent vertices are assigned the same color.

21.

22.



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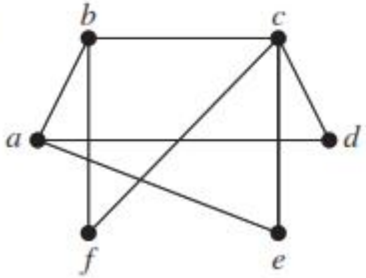

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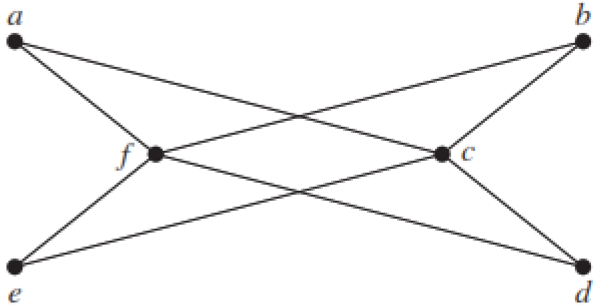

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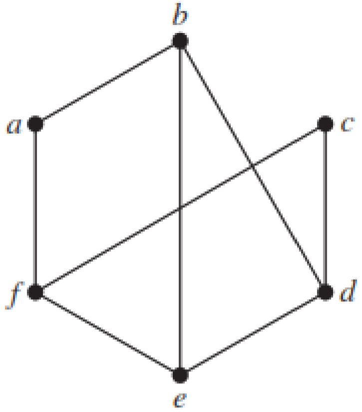
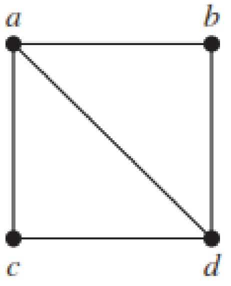
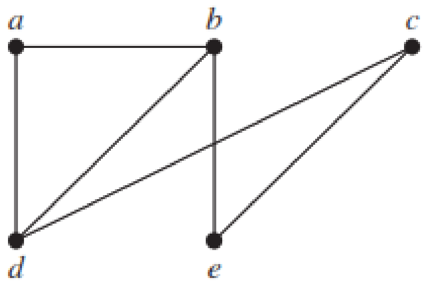
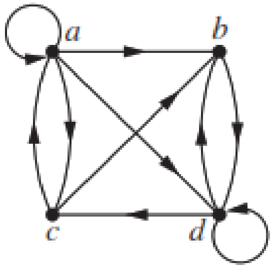
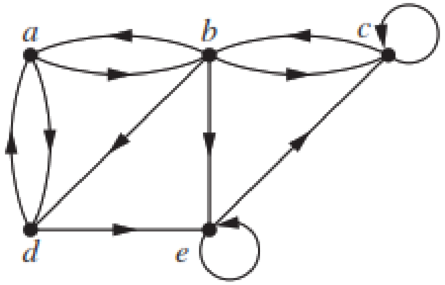


1. 

2. 

3. 

4. 

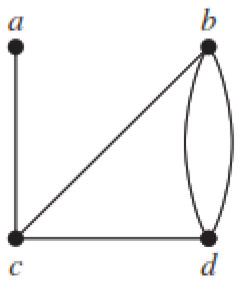
5. Represent the graph in Exercise 1 with an adjacency matrix.

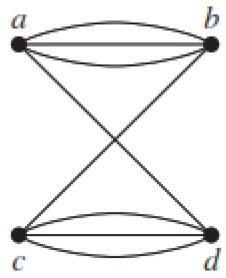
6. Represent the graph in Exercise 2 with an adjacency matrix.

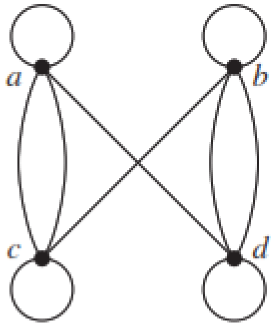
7. Represent the graph in Exercise 3 with an adjacency matrix.

8. Represent the graph in Exercise 4 with an adjacency matrix.

In Exercises 13–15 represent the given graph using an adjacency matrix.

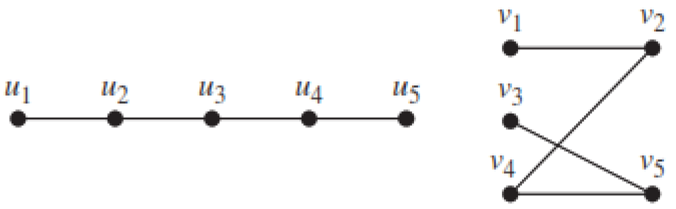
13. 

14. 

15. 

In Exercises 38–48 determine whether the given pair of graphs is isomorphic. Exhibit an isomorphism or provide a rigorous argument that none exists. For additional exercises of this kind, see Exercises 3–5 in the Supplementary Exercises.

38.



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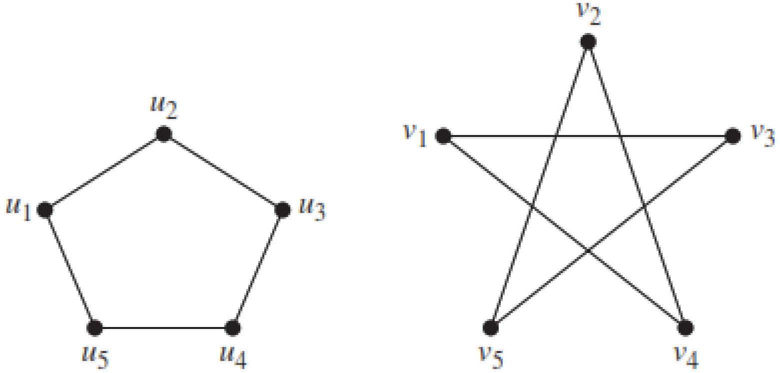

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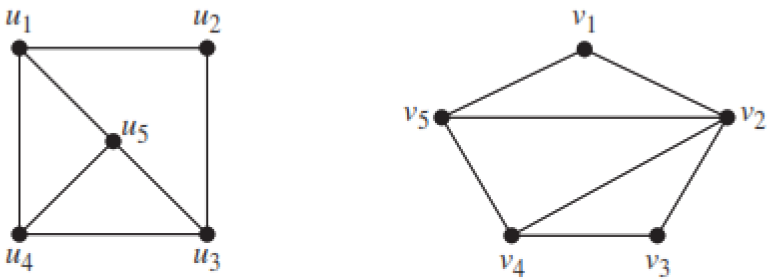

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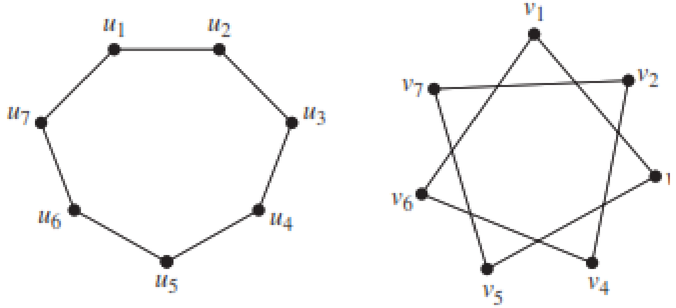

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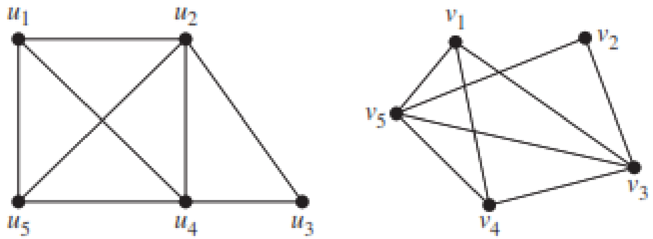

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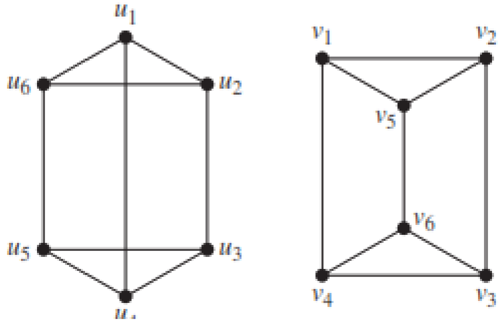

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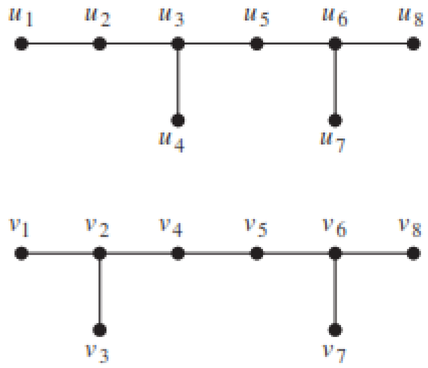

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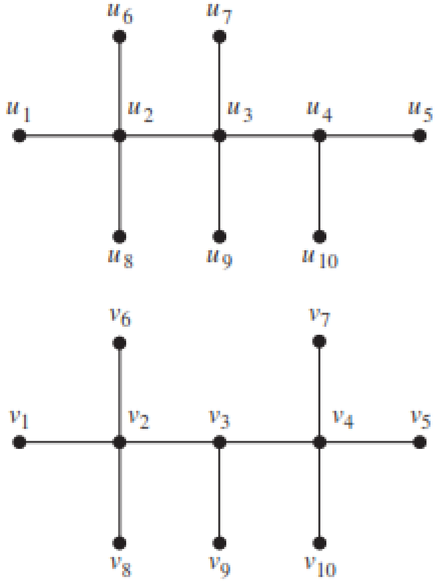
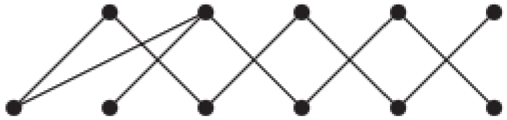
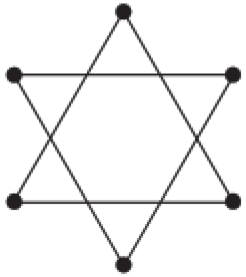


In Exercises 3–5 determine whether the given graph is connected.

3. 

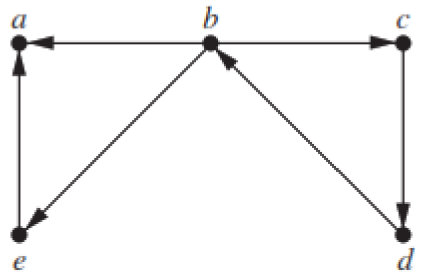
4. 

5. 

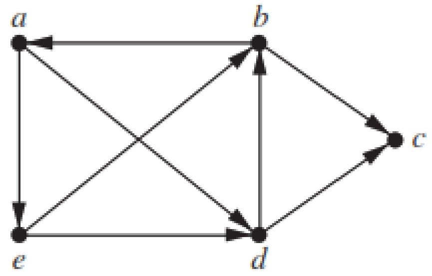
6. How many connected components does each of the graphs in Exercises 3–5 have? For each graph find each of its connected components.

11. Determine whether each of these graphs is strongly connected and if not, whether it is weakly connected.

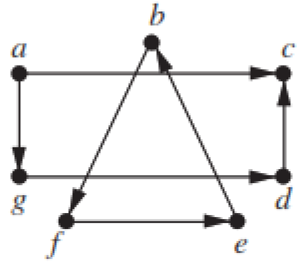
a)



b)

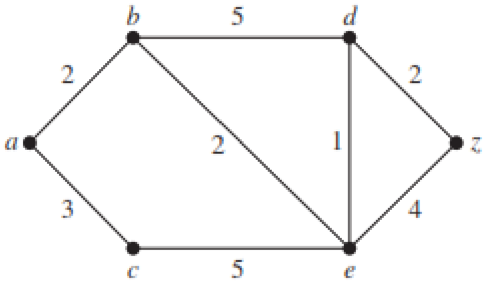


c)

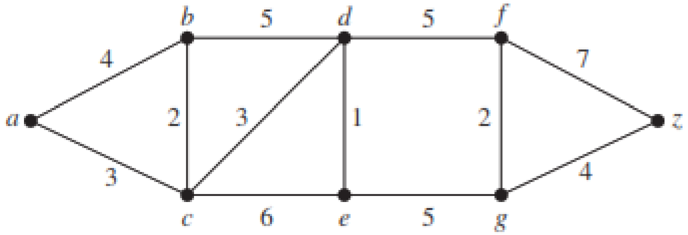


5. Find a shortest path between a and z in each of the weighted graphs in Exercises 2–4.

2.



3.



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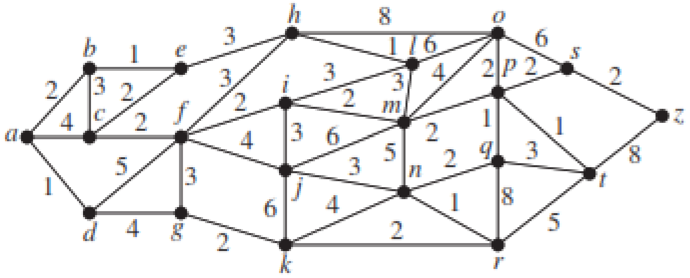
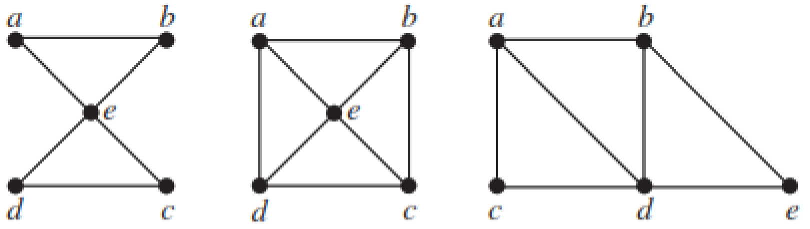


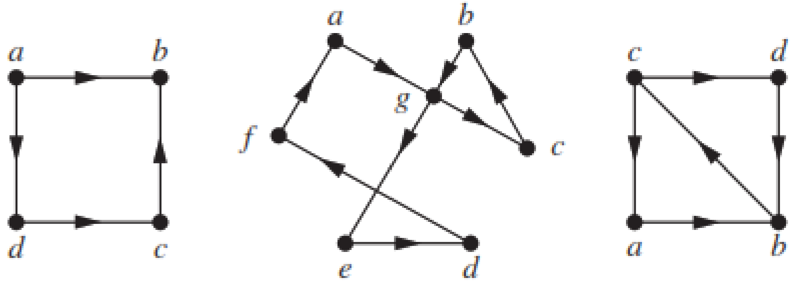
Can we travel along the edges of a graph starting at a vertex and returning to it by traversing each edge of the graph exactly once? Similarly, can we travel along the edges of a graph starting at a vertex and returning to it while visiting each vertex of the graph exactly once? Although

An *Euler circuit* in a graph G is a simple circuit containing every edge of G. An *Euler path* in G is a simple path containing every edge of G.



G_1 G_2 G_3

Solution: The graph G_1 has an Euler circuit, for example, *a, e, c, d, e, b, a*. Neither of the graphs G_2 or G_3 has an Euler circuit (the reader should verify this). However, G_3 has an Euler path, namely, *a, c, d, e, b, d, a, b*. G_2 does not have an Euler path (as the reader should verify). ◀

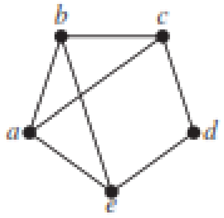


H_1 H_2 H_3

Solution: The graph H_2 has an Euler circuit, for example, *a, g, c, b, g, e, d, f, a*. Neither H_1 nor H_3 has an Euler circuit (as the reader should verify). H_3 has an Euler path, namely, *c, a, b, c, d, b*, but H_1 does not (as the reader should verify). ◀

In Exercises 1–8 determine whether the given graph has an Euler circuit. Construct such a circuit when one exists. If no Euler circuit exists, determine whether the graph has an Euler path and construct such a path if one exists.

1.



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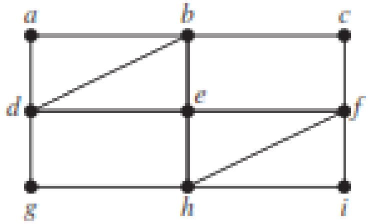
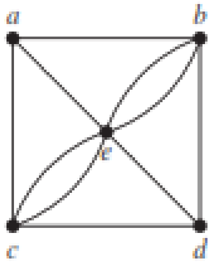


3.



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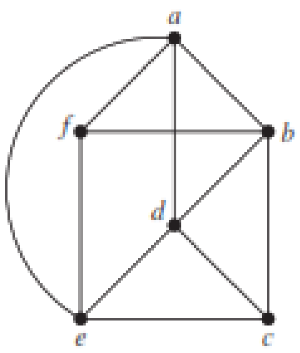
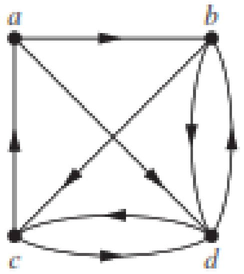


In Exercises 18–23 determine whether the directed graph shown has an Euler circuit. Construct an Euler circuit if one exists. If no Euler circuit exists, determine whether the directed graph has an Euler path. Construct an Euler path if one exists.

18.



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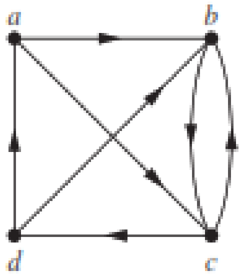

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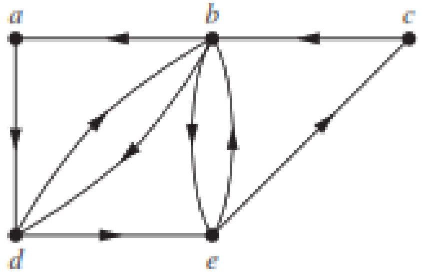
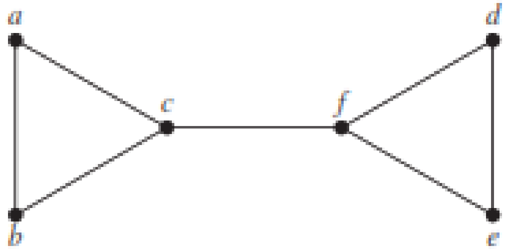


In Exercises 30–36 determine whether the given graph has a Hamilton circuit. If it does, find such a circuit. If it does not, give an argument to show why no such circuit exists.

30.



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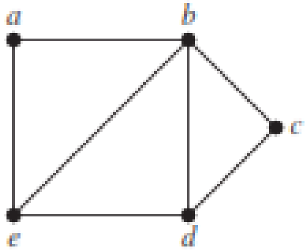

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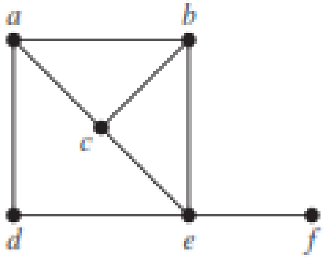

33.