

## **KINEMATICS**

more...









#### Kinematics More complicated situations

More Objects

Write an additional set of equations

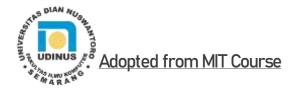
More Dimensions

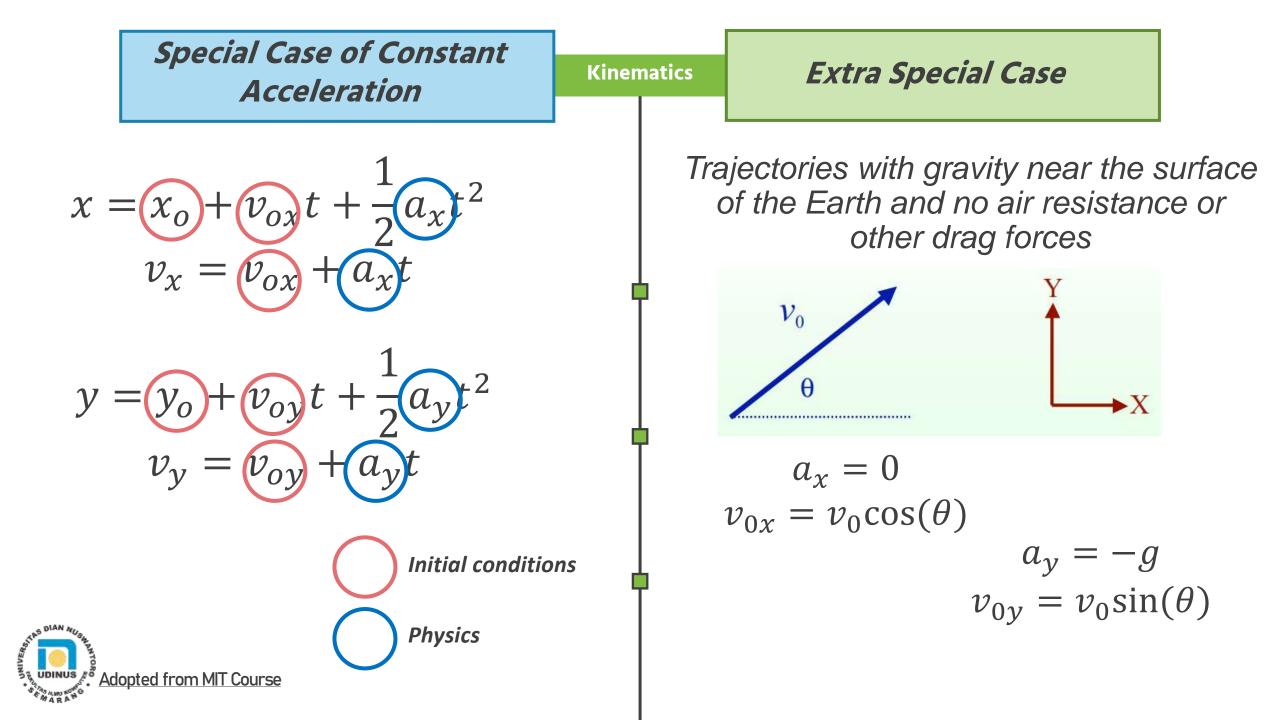
Write an additional set of equations

$$V_{x} = \frac{dx}{dt} \qquad a_{x} = \frac{dv_{x}}{dt} = \frac{d^{2}x}{dt^{2}}$$
$$V_{y} = \frac{dy}{dt} \qquad a_{y} = \frac{dv_{y}}{dt} = \frac{d^{2}y}{dt^{2}}$$

### **Vector Connections**

$$\vec{v} = \frac{d\vec{r}}{dt} \quad \vec{a} = \frac{d\vec{v}}{dt} = \frac{d^{2}\vec{r}}{dt^{2}}$$
$$|\vec{v}| = \sqrt{v_{x}^{2} + v_{y}^{2} + v_{z}^{2}}$$
$$|\vec{a}| = \sqrt{a_{x}^{2} + a_{y}^{2} + a_{z}^{2}}$$





Range of a projectile near the surface of the Earth and no air resistance or other drag forces

$$x_0 = 0$$
  $y_0 = 0$   $y_{final} = 0$   $x_{final} = Range$ 

$$Range = \frac{v_o^2 \sin(2\theta)}{g}$$

Adopted from MIT Course

You should immediately forget you ever saw this formula but remember the technique used to find it.

#### **Quadratic Equations**

$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Important property:

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Such equations can have 0, 1, or 2 solutions depending on the value of b2-4ac. Negative: 0 solutions Zero: 1 solution Positive: 2 solutions

*Warning*: Only one of the 2 solutions may be physical!

Example

#### Example 1



The bullet is fired from point O (ground) with initial velocity  $v_0 = 60m/s$  and the elevation angle from the ground is 60 degrees.  $g = 10m/s^2$  Calculate:

- a. When will the bullet hit the ground?
- b. Where the bullets will fall on the ground?
- c. What is the velocity of the bullet when it hits the ground?

#### Example

Example 1 Answer

$$y_{\text{final}} = 0$$
  

$$y = v_o Sin\theta \cdot t + \frac{1}{2}(-g)t^2$$
  

$$0 = 60Sin60^o \cdot t - \frac{1}{2}10t^2$$
  

$$0 = 30\sqrt{3} \cdot t - 5t^2$$

$$x = v_o \cos\theta.t$$
  

$$x = 60Sin60^o.6\sqrt{3}$$
  

$$x = 180\sqrt{3}m$$

C.



Bullet coordinates on the ground is ( $180\sqrt{3}, 0$ )

$$v_{y} = v_{o}Sin\theta - gt$$

$$v_{y} = 60Sin60^{o} - 10.6\sqrt{3}$$

$$v_{x} = v_{o}cos\theta$$

$$v_{y} = -30\sqrt{3}m/s$$

$$v_{x} = 60Sin60^{o}$$

$$v_{x} = 30m/s$$

 $t = 6\sqrt{3}s$ 

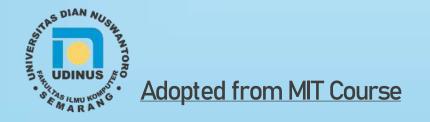
$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{30^2 + (-30\sqrt{3})^2}$$
$$v = 60m/s$$

#### **SUMMARY**

1. Study special cases (like range of a projectile) but understand the assumptions that go into all formulas

 Position, velocity, and acceleration are ALL vectors and need to be manipulated using either arrows (qualitative) or components (quantitative)

3. Directions (or signs in 1D) of position, velocity, and acceleration can all be different



# THANK YOU