Odds and ends

Key Derivation

Deriving many keys from one

Typical scenario. a single source key (SK) is sampled from:

- Hardware random number generator
- A key exchange protocol (discussed later)

Need many keys to secure session:

- unidirectional keys; multiple keys for nonce-based CBC.

Goal: generate many keys from this one source key

SK —KDF→ k_1, k_2, k_3, …

When source key is uniform

F: a PRF with key space K and outputs in $\{0,1\}^n$

Suppose source key SK is uniform in K

- Define Key Derivation Function (KDF) as:

KDF(SK, CTX, L) :=

F(SK, (**CTX II 0**)) II F(SK, (**CTX II 1**)) II ⋯ II F(SK, (**CTX II L**))

CTX: a string that uniquely identifies the application

KDF(SK, CTX, L) :=

$F(SK, (CTX \| 0)) \| F(SK, (CTX \| 1)) \| \cdots \| F(SK, (CTX \| L))$

What is the purpose of CTX?

- ◯ Even if two apps sample same SK they get indep. keys
- ◯ It's good practice to label strings with the app. name
- ◯ It serves no purpose
- ◯

What if source key is not uniform?

Recall: PRFs are pseudo random only when key is uniform in K

- SK not uniform ⇒ PRF output may not look random

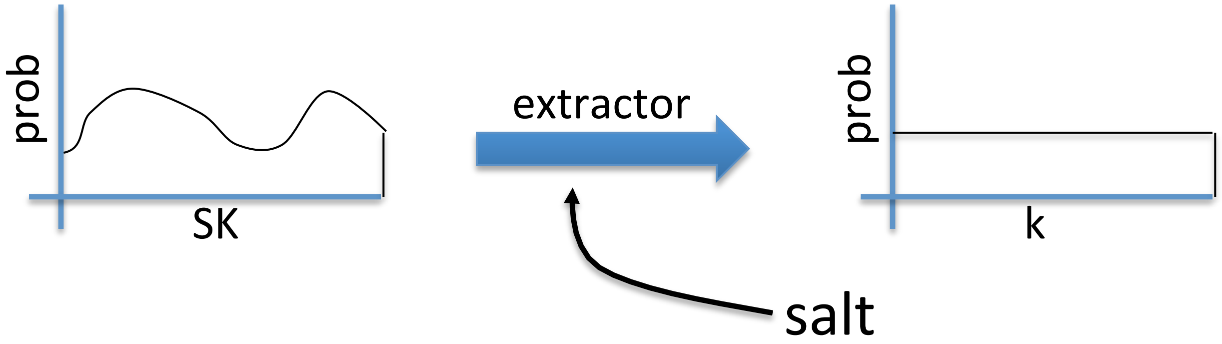
Source key often not uniformly random:

- Key exchange protocol: key uniform in some subset of K

- Hardware RNG: may produce biased output

Extract-then-Expand paradigm

Step 1: **extract** pseudo-random key k from source key SK



salt: a fixed non-secret string chosen at random

step 2: **expand** k by using it as a PRF key as before

HKDF: a KDF from HMAC

Implements the extract-then-expand paradigm:

- extract: use **k ⟵ HMAC(salt, SK)**

- Then expand using HMAC as a PRF with key **k**

Password-Based KDF (PBKDF)

Deriving keys from passwords:

- Do not use HKDF: passwords have insufficient entropy
- Derived keys will be vulnerable to dictionary attacks

(more on this later)

PBKDF defenses: **salt** and a **slow hash function**

Standard approach: **PKCS#5** (PBKDF1)

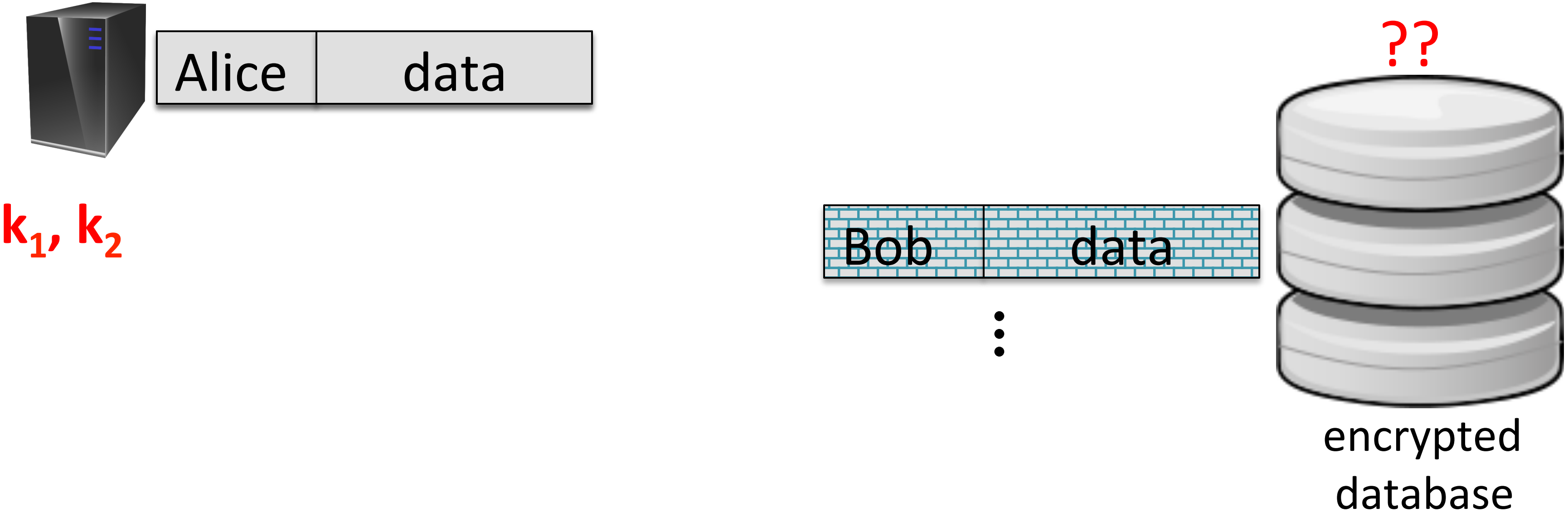
$H^{(c)}$(pwd ll salt): iterate hash function c times

End of Segment

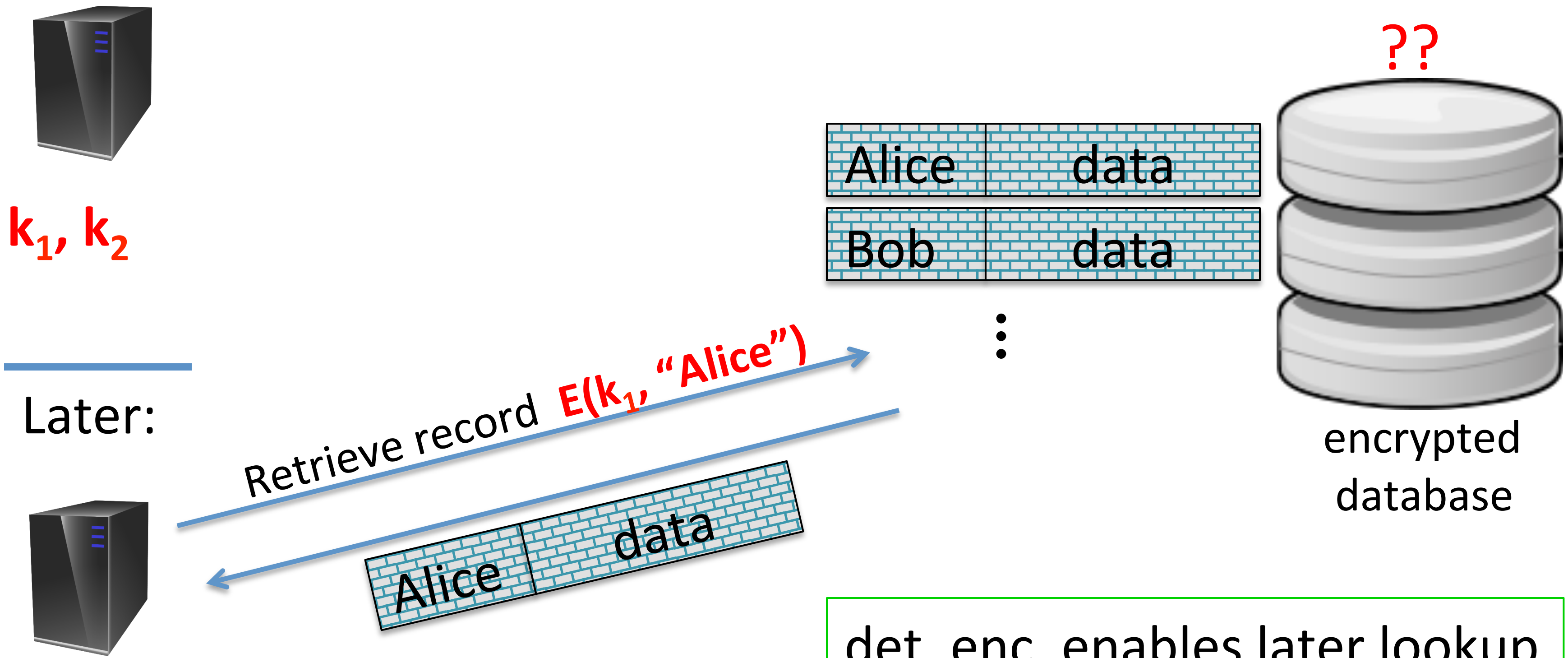
Odds and ends

Deterministic Encryption

The need for det. Encryption (no nonce)



The need for det. Encryption (no nonce)

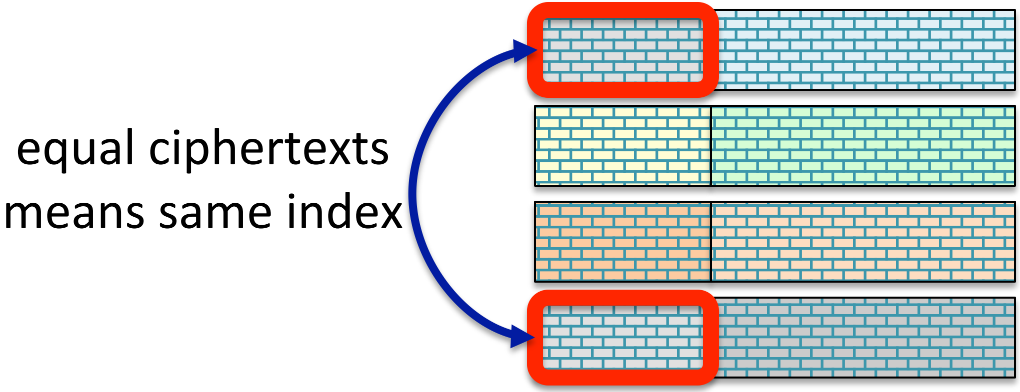


det. enc. enables later lookup

Problem: det. enc. cannot be CPA secure

The problem: attacker can tell when two ciphertexts
encrypt the same message $\Rightarrow$ leaks information

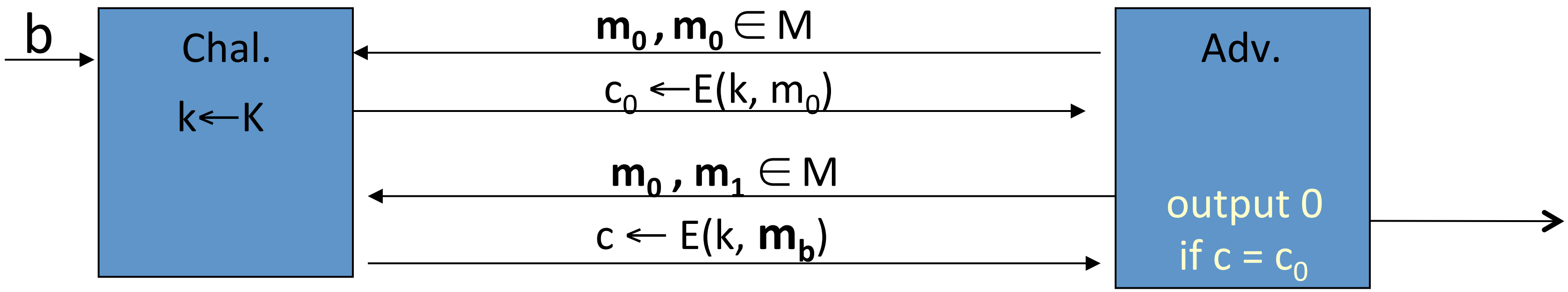
Leads to significant attacks when message space M is small.



Problem: det. enc. cannot be CPA secure

The problem: attacker can tell when two ciphertexts encrypt the same message ⇒ leaks information

Attacker wins CPA game:



A solution: the case of unique messages

Suppose encryptor **never** encrypts same message twice:

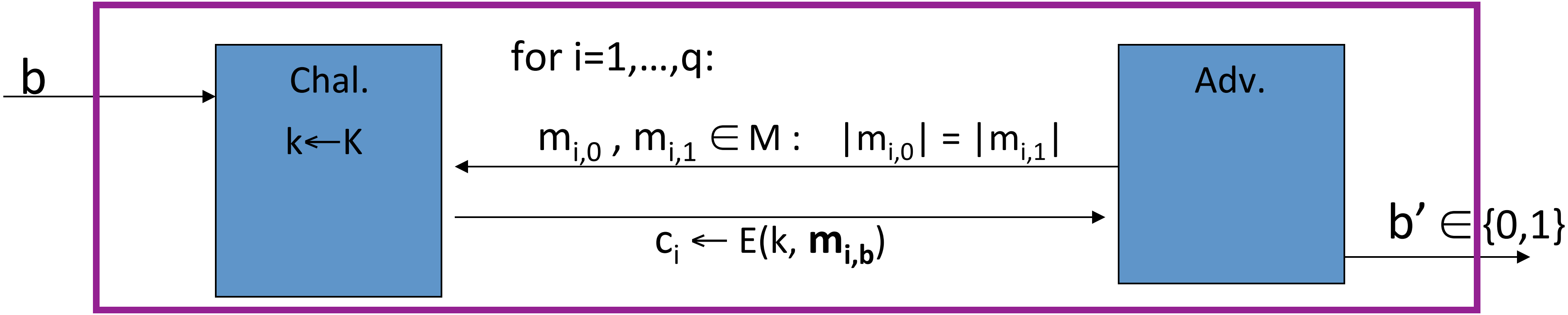
the pair (k , m) never repeats

This happens when encryptor:

- Chooses messages at random from a large msg space (e.g. keys)
- Message structure ensures uniqueness (e.g. unique user ID)

Deterministic CPA security

$\mathbb{E} = (E,D)$ a cipher defined over (K,M,C). For $b=0,1$ define EXP(b) as:



where $\mathbf{m_{1,0}, \ldots, m_{q,0}}$ are distinct and $\mathbf{m_{1,1}, \ldots, m_{q,1}}$ are distinct

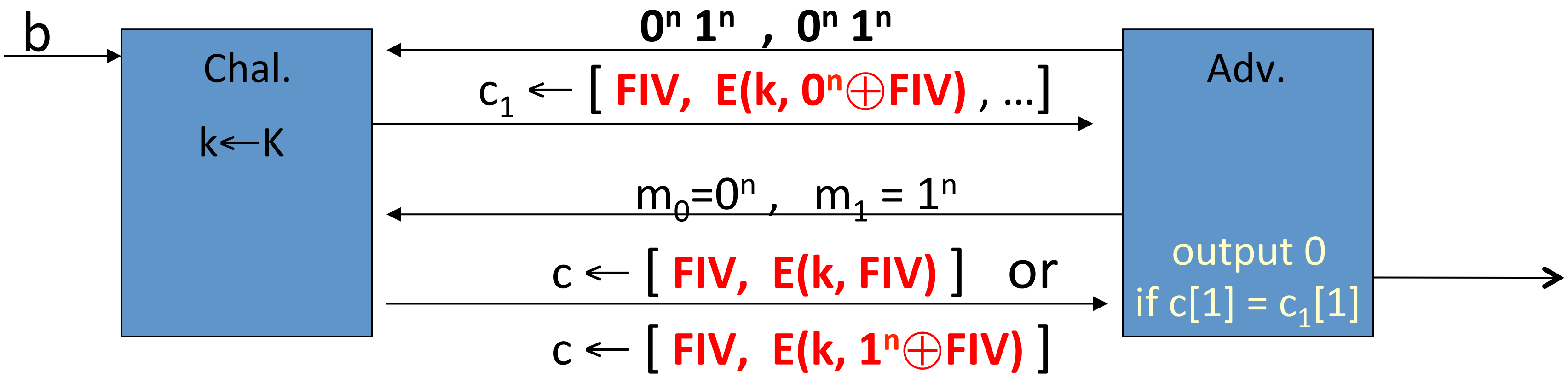
Def: $\mathbb{E}$ is **sem. sec. under det. CPA** if for all efficient A:

$$\mathrm{Adv_{dCPA}}[A,\mathbb{E}] = \left| \Pr[\mathrm{EXP}(0)=1] - \Pr[\mathrm{EXP}(1)=1] \right|$$ is negligible.

A Common Mistake

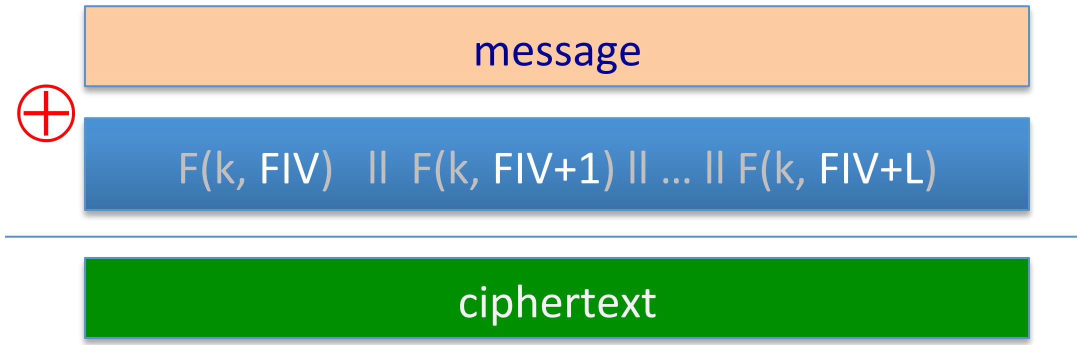
CBC with <u>fixed IV</u> is not det. CPA secure.

Let $E: K \times \{0,1\}^n \longrightarrow \{0,1\}^n$ be a secure PRP used in CBC

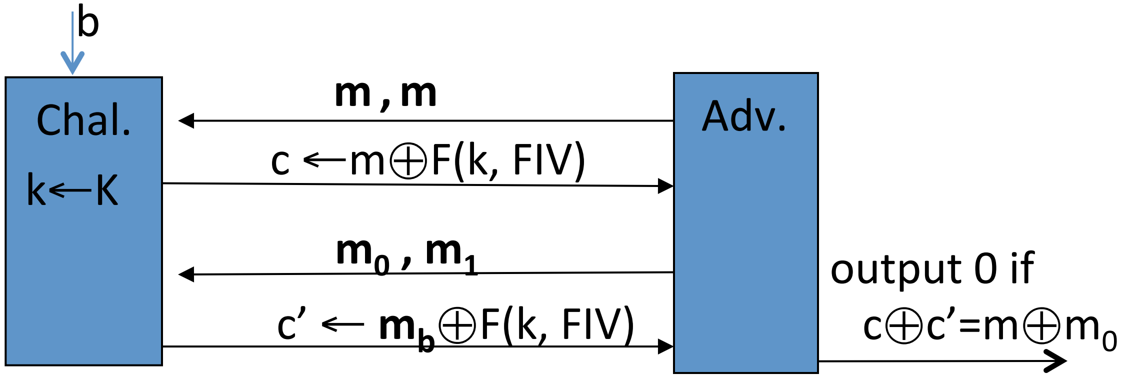


Leads to significant attacks in practice.

Is counter mode with a fixed IV det. CPA secure?



- Yes
- No
- It depends
-



End of Segment

Odds and ends

Deterministic Encryption Constructions: SIV and wide PRP

Deterministic encryption

Needed for maintaining an encrypted database index

- Lookup records by encrypted index

Deterministic CPA security:

- Security if never encrypt same message twice using same key:

  the pair (key , msg) is unique

Formally: we defined deterministic CPA security game

Construction 1: Synthetic IV (SIV)

Let (E, D) be a CPA-secure encryption. **E(k, m ; r) ⟶ c**

Let $F: K \times M \longrightarrow R$ be a secure PRF

Define: $\mathbf{E_{det}((k_1,k_2), m) =}$ $\begin{cases} r \leftarrow F(k_1, m) \\ c \leftarrow E(k_2, m ; r) \\ \text{output } r \end{cases}$

Thm: $\mathbf{E_{det}}$ is sem. sec. under det. CPA .

Proof sketch: distinct msgs. ⇒ all r's are indist. from random

Well suited for messages longer than one AES block (16 bytes)

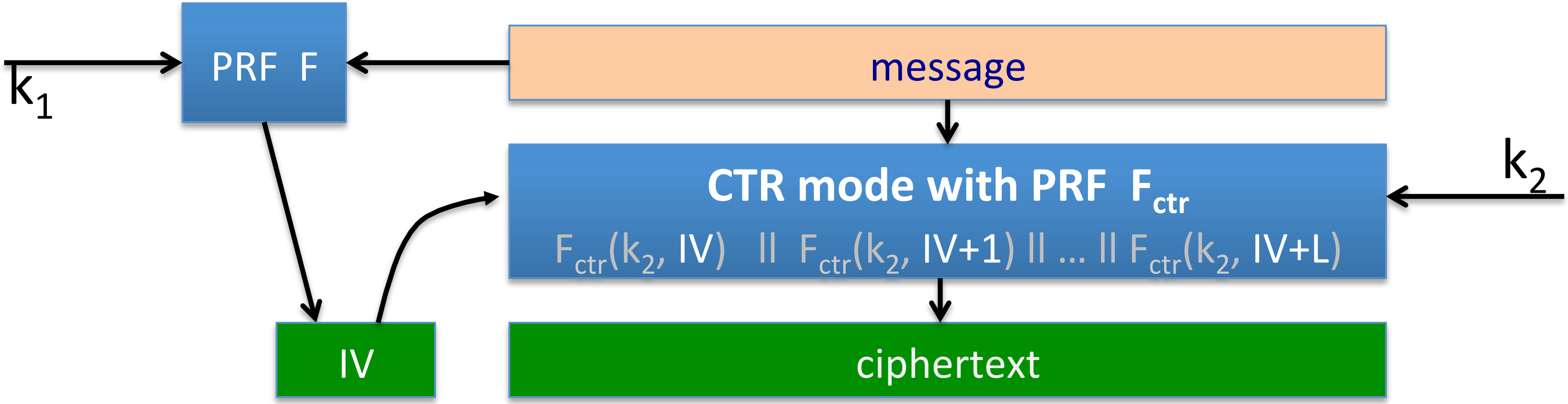
Ensuring ciphertext integrity

Goal: det. CPA security and ciphertext integrity

⇒ **DAE: deterministic authenticated encryption**

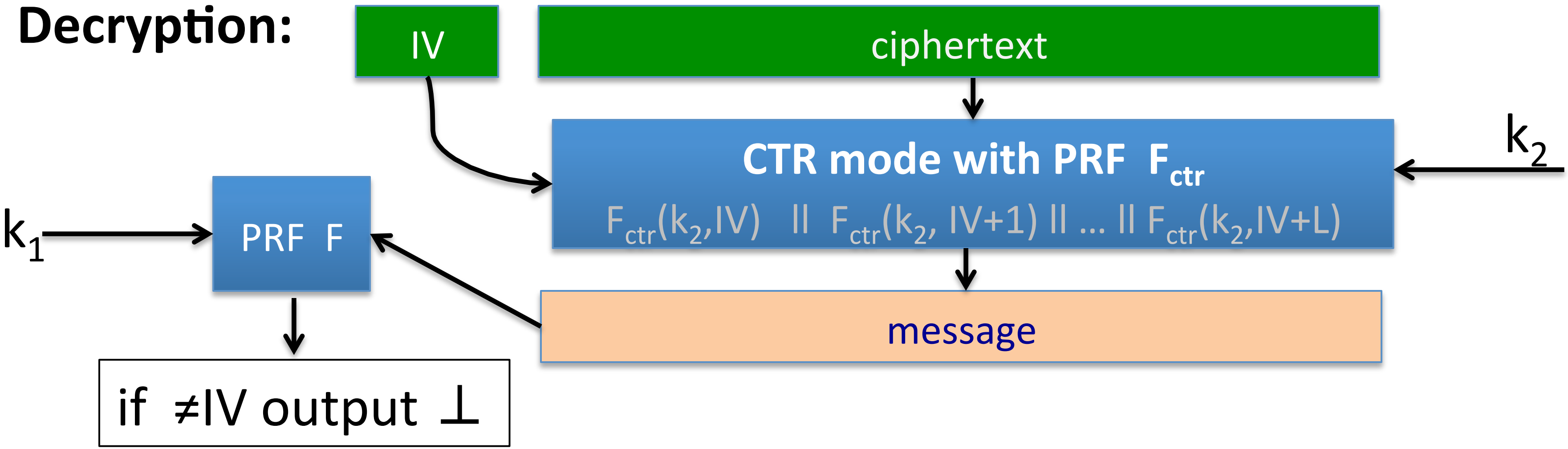
Consider a SIV special case: SIV-CTR

SIV where cipher is counter mode with rand. IV



Det. Auth. Enc. (DAE) for free

Decryption:



Thm: if F is a secure PRF and CTR from F_{ctr} is CPA-secure then SIV-CTR from F, F_{ctr} provides DAE

Construction 2: just use a PRP

Let (E, D) be a secure PRP. $E: K \times X \longrightarrow X$

Thm: (E,D) is sem. sec. under det. CPA .

Proof sketch: let $f: X \longrightarrow X$ be a truly random invertible func.

in EXP(0) adv. sees: $f(m_{1,0}), \ldots, f(m_{q,0})$

in EXP(1) adv. sees: $f(m_{1,1}), \ldots, f(m_{q,1})$

(both: q random values in X)

Using AES: Det. CPA secure encryption for 16 byte messages.

Longer messages?? Need PRPs on larger msg spaces …

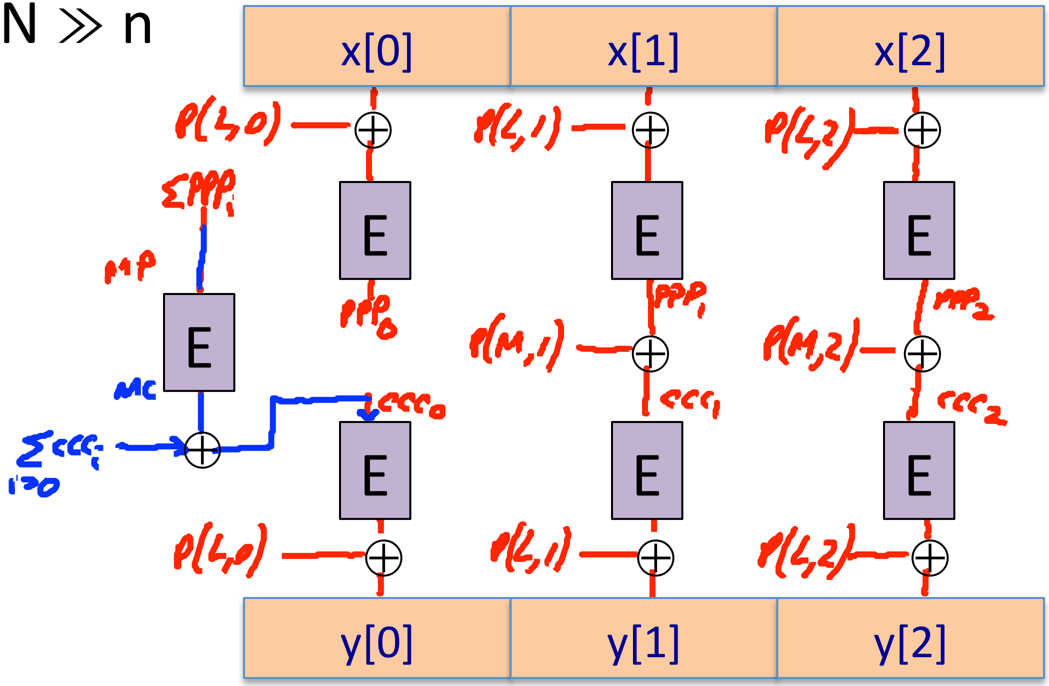
EME: constructing a wide block PRP

Let (E, D) be a secure PRP. $E: K \times \{0,1\}^n \longrightarrow \{0,1\}^n$

EME: a PRP on $\{0,1\}^N$ for $N \gg n$

Key = (K,L)

M ← MP ⊕ MC



Performance:

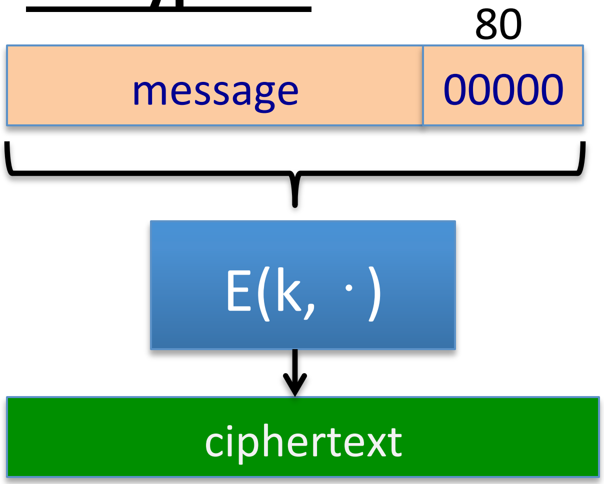
- can be 2x slower then SIV

PRP-based Det. Authenticated Enc.

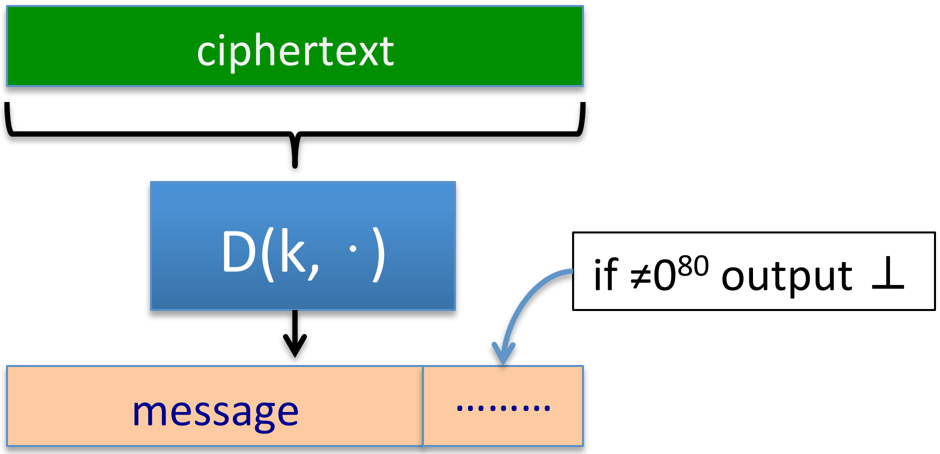
Goal: det. CPA security and ciphertext integrity

$\Rightarrow$ **DAE: deterministic authenticated encryption**

Encryption:



Decryption:

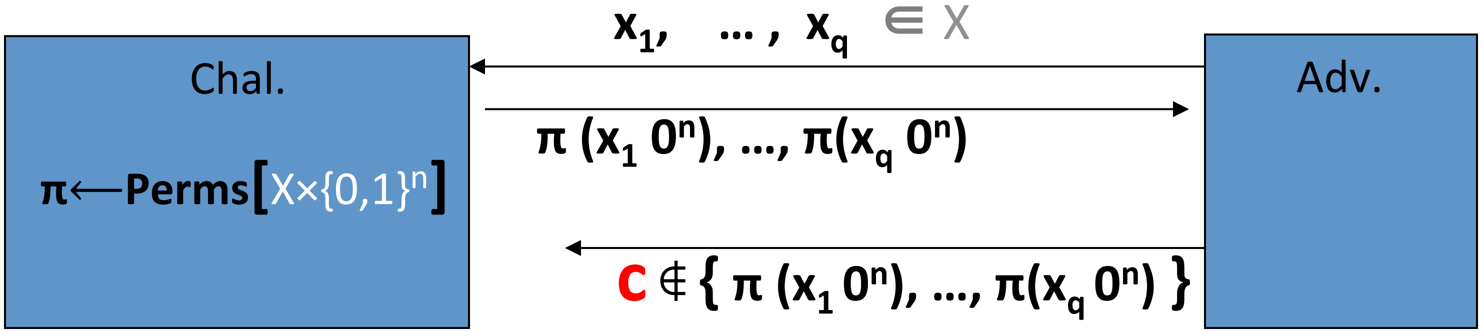


PRP-based Det. Authenticated Enc.

Let (E, D) be a secure PRP. $E: K \times (X \times \{0,1\}^n) \longrightarrow X \times \{0,1\}^n$

Thm: $1/2^n$ is negligible $\Rightarrow$ PRP-based enc. provides DAE

Proof sketch: suffices to prove ciphertext integrity



But then $\Pr[\ \text{LSB}_n(\ \pi^{-1}(c)\ ) = 0^n\ ] \le 1/2^n$

End of Segment

Odds and ends

Tweakable encryption

Disk encryption: no expansion

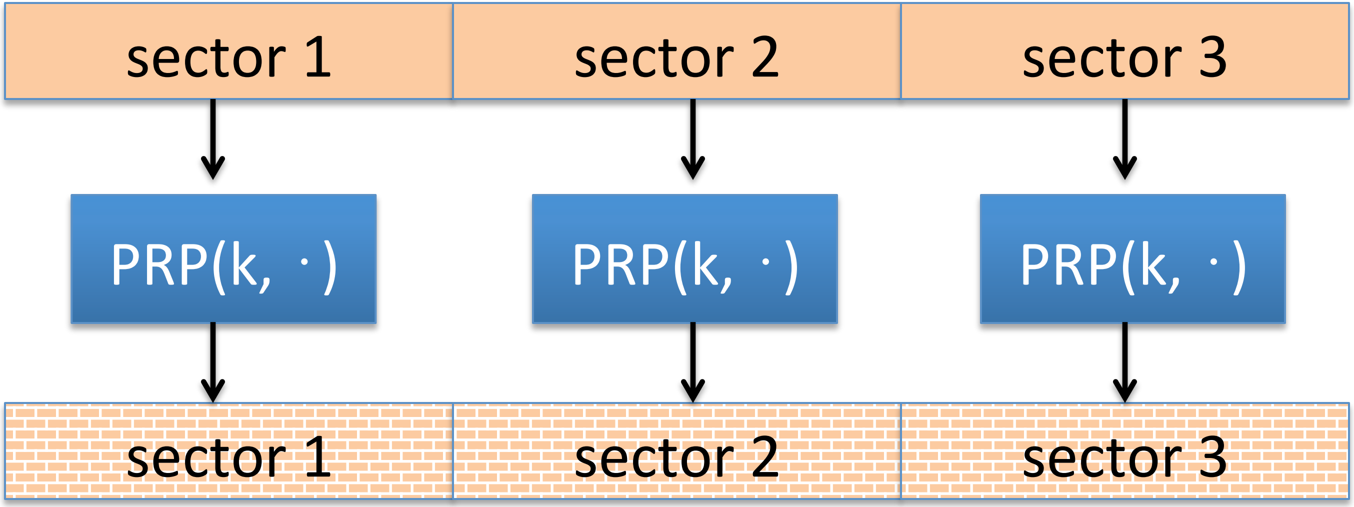
Sectors on disk are fixed size (e.g. 4KB)

⇒ encryption cannot expand plaintext (i.e. M = C)

⇒ must use deterministic encryption, no integrity

Lemma: if (E, D) is a det. CPA secure cipher with M=C
then (E, D) is a PRP.

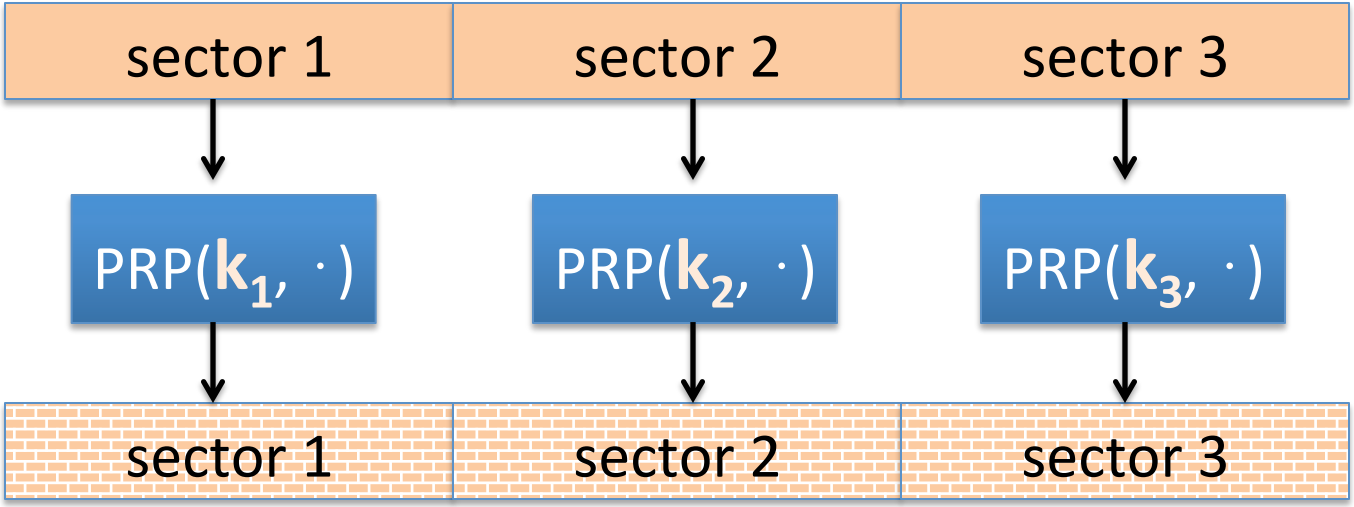
⇒ every sector will need to be encrypted with a PRP



Problem: sector 1 and sector 3 may have same content

- Leaks same information as ECB mode

Can we do better?



Avoids previous leakage problem

- … but attacker can tell if a sector is changed and then reverted

Managing keys: the trivial construction $k_t = \text{PRF}(k, t)$, t=1,…,L

Can we do better?

Tweakable block ciphers

Goal: construct **many** PRPs from a key $k \in K$.

Syntax: $\mathbf{E}, \mathbf{D}: \mathbf{K} \times \mathbf{T} \times \mathbf{X} \longrightarrow \mathbf{X}$

for every $t \in T$ and $k \leftarrow K$:

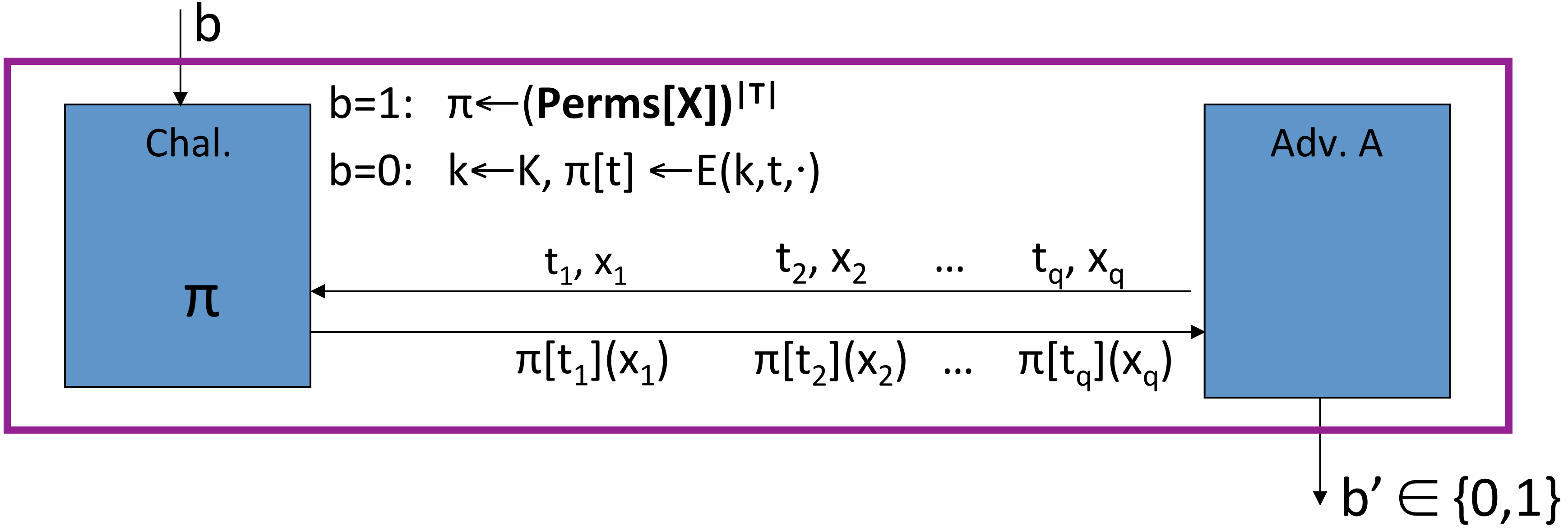
$\mathbf{E}(k, t, \cdot)$ is an invertible func. on X, indist. from random

Application: use sector number as the tweak

$\Rightarrow$ every sector gets its own independent PRP

Secure **tweakable** block ciphers

E,D: $K \times T \times X \longrightarrow X$. For b=0,1 define experiment EXP(b) as:



- Def: E is a secure tweakable PRP if for all efficient A:

$$Adv_{tPRP}[A,E] = \left| Pr[EXP(0)=1] - Pr[EXP(1)=1] \right| \text{ is negligible.}$$

Example 1: the trivial construction

Let (E,D) be a secure PRP, E: $K \times X \longrightarrow X$.

- The trivial tweakable construction: (suppose K = X)

$$E_{tweak}(k, t, x) = E\big(E(k, t),\ x \big)$$

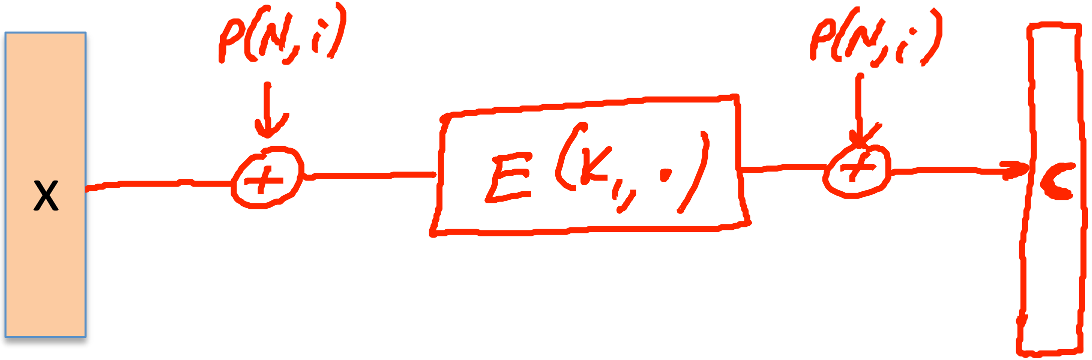
⇒ to encrypt n blocks need 2n evals of E(.,.)

2. the XTS tweakable block cipher [R'04]

Let (E,D) be a secure PRP, E: $K \times \{0,1\}^n \longrightarrow \{0,1\}^n$.

- XTS: $E_{tweak}\big((k_1,k_2),\ (t,i),\ x \big) =$

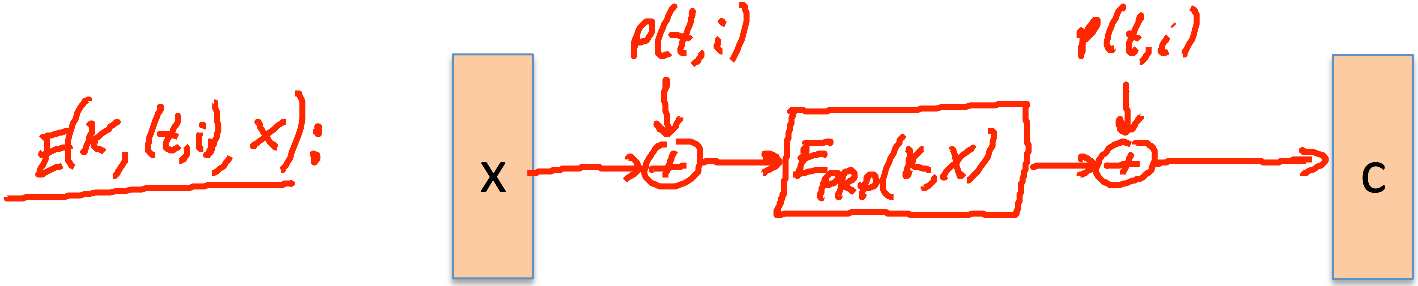
$N \longleftarrow E(k_2,\ t)$



⇒ to encrypt n blocks need n+1 evals of E(.,.)

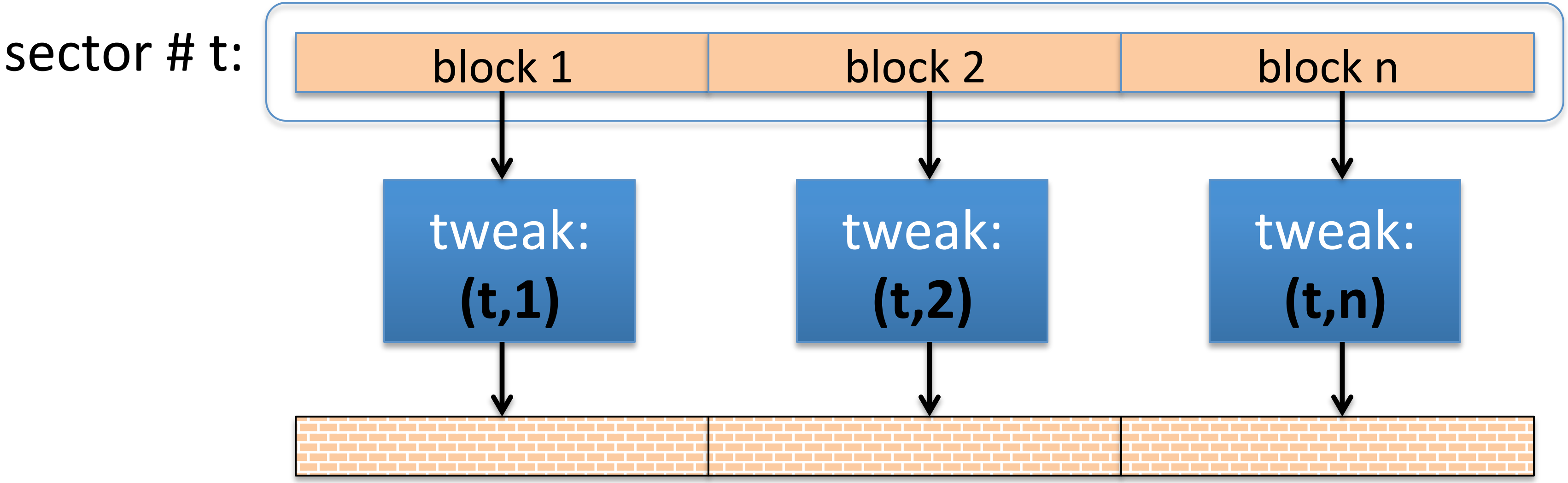
Is it necessary to encrypt the tweak before using it?

That is, is the following a secure tweakable PRP?



- Yes, it is secure
- No: $E(k, (t,1), P(t,2)) \oplus E(k, (t,2), P(t,1)) = P(t,1)$
- No: $E(k, (t,1), P(t,1)) \oplus E(k, (t,2), P(t,2)) = P(t,1) \oplus P(t,2)$
- No: $E(k, (t,1), P(t,1)) \oplus E(k, (t,2), P(t,2)) = 0$

Disk encryption using XTS



- note: block-level PRP, not sector-level PRP.
- Popular in disk encryption products:

Mac OS X-Lion, TrueCrypt, BestCrypt, …

Summary

- Use tweakable encryption when you need many independent PRPs from one key
- XTS is more efficient than the trivial construction
  - Both are narrow block: 16 bytes for AES
- EME (previous segment) is a tweakable mode for wide block
  - 2x slower than XTS

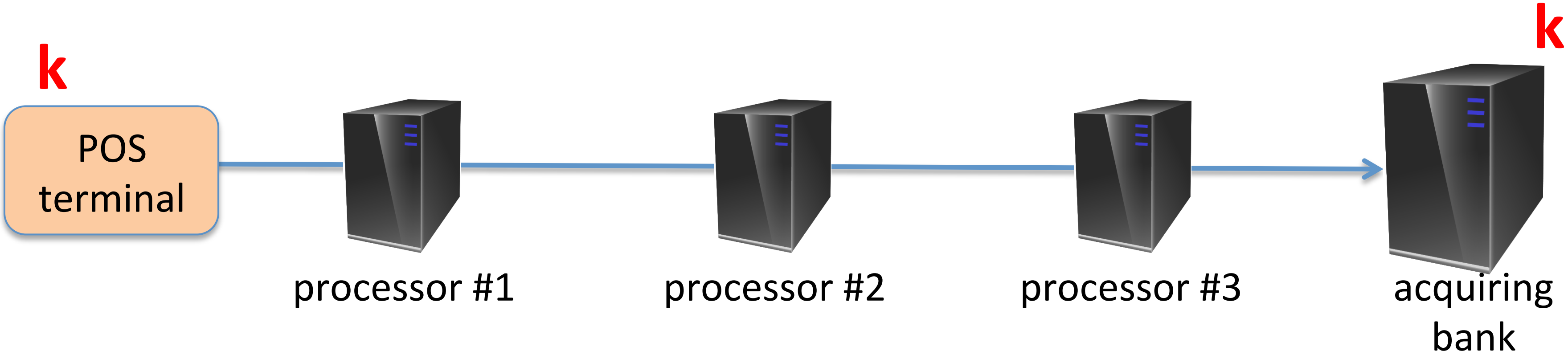
End of Segment

Odds and ends

Format preserving encryption

Encrypting credit card numbers

Credit card format: **bbbb bbnn nnnn nnnc** (≈ 42 bits)



Goal: end-to-end encryption

Intermediate processors expect to see a credit card number

⇒ encrypted credit card should look like a credit card

Format preserving encryption (FPE)

This segment: given $0 < s \leq 2^n$, build a PRP on $\{0,\ldots,s-1\}$

from a secure PRF **$F: K \times \{0,1\}^n \longrightarrow \{0,1\}^n$** (e.g. AES)

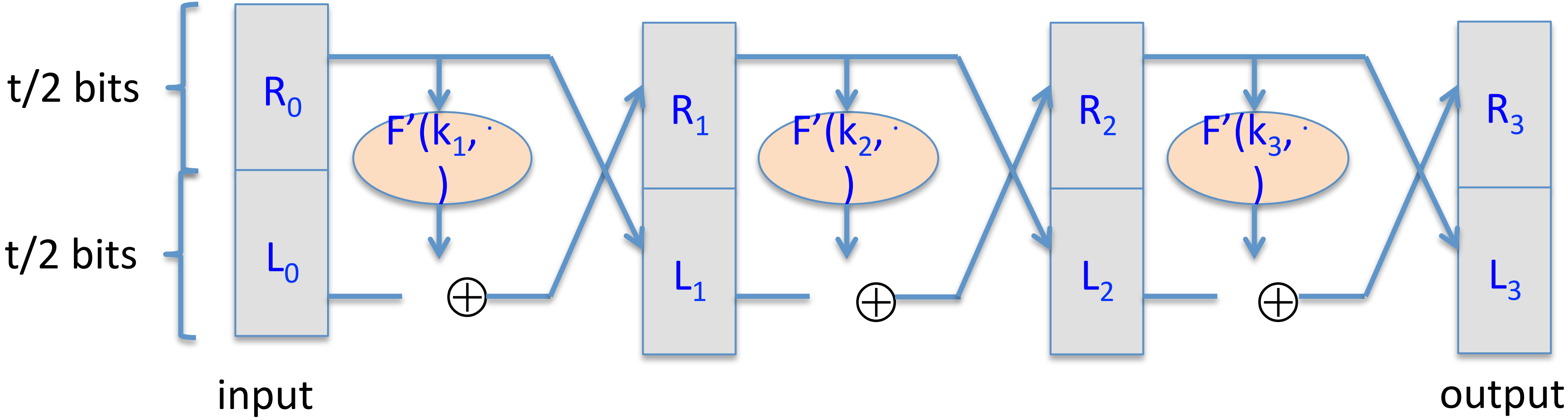
Then to encrypt a credit card number: (s = total # credit cards)

1. map given CC# to $\{0,\ldots,s-1\}$
2. apply PRP to get an output in $\{0,\ldots,s-1\}$
3. map output back a to CC#

Step 1: from $\{0,1\}^n$ to $\{0,1\}^t$ (t<n)

Want PRP on $\{0,\ldots,s-1\}$. Let t be such that $2^{t-1} < s \le 2^t$.

Method: Luby-Rackoff with $F': K \times \{0,1\}^{t/2} \longrightarrow \{0,1\}^{t/2}$ (truncate F)



(better to use 7 rounds a la Patarin, Crypto'03)

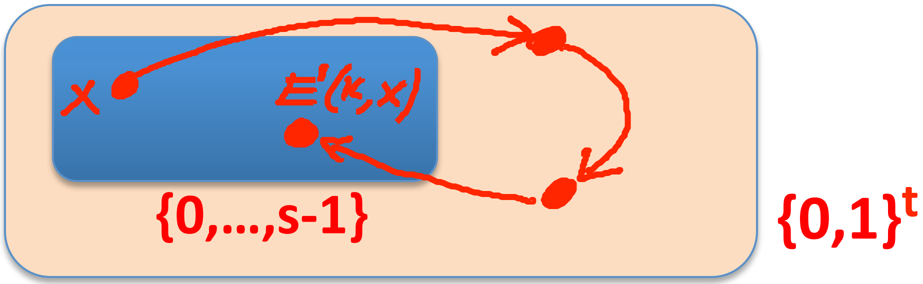
Step 2: from $\{0,1\}^t$ to $\{0,\ldots,s\text{-}1\}$

Given PRP $(E,D): K \times \{0,1\}^t \longrightarrow \{0,1\}^t$

we build $(E',D'): K \times \{0,\ldots,s\text{-}1\} \longrightarrow \{0,\ldots,s\text{-}1\}$

E'(k, x): on input $x \in \{0,\ldots,s\text{-}1\}$ do:

$y \leftarrow x$; do { $y \leftarrow E(k, y)$ } until $y \in \{0,\ldots,s\text{-}1\}$; output y



Expected # iterations: 2

Security

Step 2 is tight: $\forall$ A $\exists$ B: $\text{PRP}_{\text{adv}}[A,E] = \text{PRP}_{\text{adv}}[B,E']$

Intuition: $\forall$ sets $Y \subseteq X$, applying the transformation to a random perm. $\pi: X \longrightarrow X$

gives a random perm. $\pi': Y \longrightarrow Y$

Step 1: same security as Luby-Rackoff construction

(actually using analysis of Patarin, Crypto'03)

note: no integrity

Further reading

- Cryptographic Extraction and Key Derivation: The HKDF Scheme.
  H. Krawczyk, Crypto 2010
- Deterministic Authenticated-Encryption:
  A Provable-Security Treatment of the Keywrap Problem.
  P. Rogaway, T. Shrimption, Eurocrypt 2006
- A Parallelizable Enciphering Mode. S. Halevi, P. Rogaway, CT-RSA 2004
- Efficient Instantiations of Tweakable Blockciphers and Refinements to Modes OCB and PMAC. P. Rogaway, Asiacrypt 2004
- How to Encipher Messages on a Small Domain:
  Deterministic Encryption and the Thorp Shuffle.
  B. Morris, P. Rogaway, T. Stegers, Crypto 2009

End of Segment