

Symmetric and Antisymmetric Relations



explain deep please about antisymmetric, symmetric binary relation

In mathematics, particularly in set theory and logic, binary relations are ways of describing how elements from one set are related to elements of another set. A **binary relation** R on a set A is a set of ordered pairs of elements from A , such that for any pair (a, b) , we say that "a is related to b" if $(a, b) \in R$.

For example, let's consider a set $A = \{1, 2, 3\}$, and let's define a binary relation R on A as follows:

$$R = \{(1, 2), (2, 3), (1, 3)\}$$

This means that 1 is related to 2, 2 is related to 3, and 1 is related to 3 in the context of the relation R .

Symmetric Binary Relation:

A **symmetric** binary relation on a set A is defined as follows: if an element a is related to an element b (i.e., $(a, b) \in R$), then b must also be related to a (i.e., $(b, a) \in R$).

Formal definition:

A binary relation R on a set A is **symmetric** if:

$$\forall a, b \in A, (a, b) \in R \implies (b, a) \in R$$

Example:

Let $A = \{1, 2, 3\}$, and define R as follows:

$$R = \{(1, 2), (2, 1), (2, 3), (3, 2)\}$$

In this case, for every pair $(a, b) \in R$, the reverse pair (b, a) is also in R . For example, since $(1, 2) \in R$, we also have $(2, 1) \in R$. Hence, the relation is symmetric.

Non-Symmetric Example:

Consider $A = \{1, 2, 3\}$, and let $R = \{(1, 2), (2, 3)\}$. In this case:

- $(1, 2) \in R$ but $(2, 1) \notin R$
- $(2, 3) \in R$ but $(3, 2) \notin R$

This relation is **not symmetric** because the reverse of some pairs are not included in R .

Antisymmetric Binary Relation:

A **antisymmetric** binary relation on a set A has a different property. Specifically, if an element a is related to an element b , and b is also related to a , then a must be equal to b .

Formal definition:

A binary relation R on a set A is **antisymmetric** if:

$$\forall a, b \in A, ((a, b) \in R \text{ and } (b, a) \in R) \implies a = b$$

Example:

Let $A = \{1, 2, 3\}$, and define R as follows:

$$R = \{(1, 2), (2, 3), (1, 1)\}$$

This relation is **antisymmetric** because:

- $(1, 2) \in R$ and $(2, 1) \notin R$ (no violation)
- $(2, 3) \in R$ and $(3, 2) \notin R$ (no violation)
- $(1, 1) \in R$, and since $1 = 1$, there is no violation of antisymmetry.

The key point is that whenever both (a, b) and (b, a) exist in R , it must be that $a = b$. So, if both (a, b) and (b, a) were to exist in the relation, we would be forced to conclude that $a = b$.

Non-Antisymmetric Example:

Consider $A = \{1, 2, 3\}$, and let $R = \{(1, 2), (2, 1)\}$. In this case:

- Both $(1, 2) \in R$ and $(2, 1) \in R$, but $1 \neq 2$, which violates the condition for antisymmetry.

Thus, the relation is **not antisymmetric**.

Summary of Differences:

- **Symmetric Relation:** If $(a, b) \in R$, then $(b, a) \in R$.
- **Antisymmetric Relation:** If $(a, b) \in R$ and $(b, a) \in R$, then $a = b$.

Key Points to Remember:

- A relation can be both symmetric and antisymmetric, but only in very specific cases (e.g., the identity relation where only pairs (a, a) are included).
- Symmetry means the relation is "bi-directional," while antisymmetry means that if there is a reverse relation, the two elements must be identical.

Would you like to dive deeper into how these properties interact with other types of binary relations, such as transitive relations?

