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Properties of Binary Relations

Binary relations are relationships defined between pairs of elements from a set or between two sets. These relations can have specific properties that help classify and analyze them. Below are the key properties of binary relations:

1. Reflexive

A relation $ R $ on a set $ A $ is reflexive if every element in $ A $ is related to itself. Formally:

$$\forall x \in A, (x, x) \in R$$

Example: The equality relation ($ = $) on integers is reflexive because every integer is equal to itself [1] [2].

2. Irreflexive

A relation $ R $ is irreflexive if no element in $ A $ is related to itself. Formally:

$$\forall x \in A, (x, x) \notin R$$

Example: The "less-than" relation ($ < $) on integers is irreflexive because no integer is less than itself [2] [3].

3. Symmetric

A relation $ R $ is symmetric if whenever an element $ x $ is related to $ y $, then $ y $ is also related to $ x $. Formally:

$$\forall x, y \in A, (x, y) \in R \implies (y, x) \in R$$

Example: The "is a sibling of" relation is symmetric because if $ x $ is a sibling of $ y $, then $ y $ is also a sibling of $ x $ [2] [3].

4. Antisymmetric

A relation $ R $ is antisymmetric if whenever both $ (x, y) \in R $ and $ (y, x) \in R $, it must be the case that $ x = y $. Formally:

$$\forall x, y \in A, (x, y) \in R \land (y, x) \in R \implies x = y$$

Example: The "less-than-or-equal-to" ($ \leq $) relation on integers is antisymmetric because if $ x \leq y $ and $ y \leq x $, then $ x = y $ [2] [3].

5. Transitive

A relation $ R $ is transitive if whenever an element $ x $ is related to $ y $, and $ y $ is related to $ z $, then $ x $ is also related to $ z $. Formally:

$$\forall x, y, z \in A, (x, y) \in R \land (y, z) \in R \implies (x, z) \in R$$

Example: The "less-than" ($ < $) relation on integers is transitive because if $ x < y $ and $ y < z $, then $ x < z $[2] [3].

6. Equivalence Relation

An equivalence relation satisfies reflexivity, symmetry, and transitivity. It partitions the set into equivalence classes where all elements within a class are mutually related[1] [2].

7. Partial Order

A partial order satisfies reflexivity, antisymmetry, and transitivity. It defines a hierarchy or ordering among elements[2] [3].

Understanding these properties enables deeper analysis of binary relations in mathematics and computer science applications such as database theory, graph theory, and discrete mathematics.

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1. https://www.cs.odu.edu/~toida/nerzic/content/relation/property/property.html
2. http://web.cecs.pdx.edu/~harry/discrete/slides/Section4.1.pdf
3. https://eng.libretexts.org/Courses/Fresno_City_College/Discrete_Mathematics_for_Computer_Science_(Jin_He)/03:_Functions_and_Binary_Relations/3.02:_Binary_Relations