Definition of Factorial

The factorial of a non-negative integer $ n $, denoted as $ n! $, is the product of all positive integers less than or equal to $ n $. Mathematically, it is defined as:

$$n! = n \times (n - 1) \times (n - 2) \times \ldots \times 1$$

By convention, the factorial of zero is defined as:

$$0! = 1$$

This definition allows for compact expressions in many areas of mathematics and ensures consistency in combinatorial formulas[1] [2] [3].

Examples

- $ 3! = 3 \times 2 \times 1 = 6 $
- $ 5! = 5 \times 4 \times 3 \times 2 \times 1 = 120 $
- $ 0! = 1 $[4] [1] [5]

Applications

Factorials are widely used in:

- **Permutations and Combinations:** Calculating the number of ways objects can be arranged or selected[4] [6] [7].
- **Binomial Theorem:** Appearing in binomial coefficients as part of the expansion formula[1] [2].
- **Probability and Statistics:** Used in various probability formulas and distributions[7].
- **Algebra and Calculus:** Factorials appear in Taylor series expansions and other mathematical sequences[2].

Notation

The factorial function is represented by an exclamation point following the number, for example, $ n! $[4] [1] [5].

Recursive Definition

Factorials can also be defined recursively:

$$n! = n \times (n - 1)!$$

with the base case $ 0! = 1 $[3] [7].

Summary Table

n	$n!$
0	1
1	1
2	2
3	6
4	24
5	120
6	720
7	5040

Factorials are fundamental in mathematics, especially in counting and arrangement problems, and have extensions to non-integer values via the gamma function [1] [2].

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1. https://www.britannica.com/science/factorial
2. https://corporatefinanceinstitute.com/resources/data-science/factorial/
3. https://www.freecodecamp.org/news/what-is-a-factorial/
4. https://byjus.com/maths/factorial/
5. https://www.mathsisfun.com/definitions/factorial.html
6. https://study.com/learn/lesson/how-factorials-work.html
7. https://www.geeksforgeeks.org/factorial/